SOLUTION OF MAGNETOACOUSTIC HEATING PROBLEMS FOR THIN SHELLS

A. L. Radovinskii

Izv. AN SSSR. Mekhanika Tverdogo Tela,
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In this paper, we propose a simple method for calculating the evolution of heat in thin elastic shells in a harmonic magnetic field. We take account of heat evolution associated with eddy-current losses, and also the evolution that arises as a result of loss to internal friction upon mechanical vibration of the shell in response to ponderomotive forces. Using an example, we demonstrate that at resonant frequencies, heat evolution of the second type, beginning at certain magnetic induction levels of the specified magnetic field, is predominant. This is in agreement with the findings of [1,2].

The initial equations are taken in the form of [3]. Internal friction is taken into account by introducing the complex elastic modulus of [4].

1. SOLUTION METHOD

We will investigate the problem in the following sequence. We obtain the solution of the equations of [5] of the electrodynamics of thin shells (the factors exp(iωt) are discarded):

$$\gamma\Delta_s F + i\omega f_3 = -i\omega B_3, \quad \Delta\Phi = 0$$
$$f_3 = (\partial\Phi/\partial\alpha_3)^+ = (\partial\Phi/\partial\alpha_3)^-, \quad F = \Phi^+ - \Phi^- \tag{1.1}$$

We determine the magnetic pressure on the basis of the formula (the factors exp(iΩt) are dicarded; Ω = 2ω):

$$\mathbf{X} = -\mu_0^{-1}(\mathrm{grad}_s F\cdot \mathbf{i}_3)\times(\mathbf{B}+\mathbf{f}), \quad \mathbf{f} = {}^1/_2[(\mathrm{grad}\,\Phi)^+ + (\mathrm{grad}\,\Phi)^-] \tag{1.2}$$

We integrate the dynamic equations of the shells (the exp(iΩt) are discarded):

$$(1+i\delta)2EhL_h\mathbf{u} - 2\rho h\Omega^2\mathbf{u} = \mathbf{X} \tag{1.3}$$

Then the average loss power can be determined via the formulas

$$\langle P\rangle = \langle P_e\rangle + \langle P_m\rangle$$
$$\langle P_e\rangle = {}^1/_2(2h\sigma)^{-1}[\mathbf{J}\cdot\bar{\mathbf{J}}]_s = \gamma(2\mu_0)^{-1}[\mathrm{grad}_s F\cdot\overline{\mathrm{grad}_s F}]_s$$
$$\langle P_m\rangle = {}^1/_2\,\mathrm{Re}[\mathbf{x}\cdot\partial\bar{\mathbf{U}}/\partial t]_s = \Omega/2\,\mathrm{Im}[\mathbf{X}\cdot\bar{\mathbf{u}}],$$
$$\mathbf{x} = \mathbf{X}\exp(i\Omega t), \quad \mathbf{U} = \mathbf{u}\exp(i\Omega t) \tag{1.4}$$

where α_j, $\mathbf{i}_j$ (j = 1, 2, 3) are the parameters of the coordinate system associated with the center surface S of the shell (α_3 = 0 on S) and the corresponding unit vectors; B is the value on S of the magnetic induction of the source, defined in the absence of the shell (under the assumption that all of infinite three-dimensional space V exhibits vacuum properties); ω is the circular frequency of the field of the inductor; Φ is the magnetic potential (the magnetic induction of eddy currents in V is b = gradΦ, while its value on S is f); the grad and Δ operators without subscripts and with subscript s are defined in domain V and on surface S respectively; L_h is the operator of shell theory [6]; $\gamma = (2h\mu_0\sigma)^{-1}$; μ_0 is the magnetic constant; h, σ, ρ, E, δ are the half-thickness, conductivity, density, Young's modulus, and coefficient of internal friction of the shell material; for quantities denoted by $(\ldots)^\pm$, values on the faces of the mathematical cut S are taken, i.e., for α_3 = 0; $[\ldots]_s$ is the integral of the quantities in brackets over surface S; J is the linear eddy-current density [5]; the bar denotes the complex conjugate; u are the displacements of the center surface of the deformed shell.

Analytic and numerical solution methods for the three-dimensional problem of integrating the electrodynamic equations for thin shells were considered in [5-8]. It was pointed out in these studies, in particular, that for low frequency values the problem can be reduced to a two-dimensional one, referred to the parameters of the coordinates of S. In what follows, therefore, we will not dwell on solution methods for Eqs. (1.1).

Much greater mathematical difficulties arise in the stage of integrating the dynamic equations of shells (1.3). These stem primarily from the unwieldiness of operator L_h (an eighth-order operator with a small parameter in the high-order derivative). Here it seems advisable to resort to solving Eqs. (1.3) by expansion in eigensolutions of the equations of vibrations of thin elastic shells.

2. LOSSES TO INTERNAL FRICTION

Assume that $(\mathbf{V}_q, \Omega_q)$, $q=1, 2, \ldots$ are eigensolutions of the equations of free vibrations of thin elastic shells:

$$2EhL_h\mathbf{u}-2\rho h\Omega^2\mathbf{u}=0 \tag{2.1}$$

The properties of these eigensolutions were investigated in detail in [6]. It was shown in particular that if L_h is a self-adjoint operator (i.e., under idealized boundary conditions of shell theory, including widely employed hinging, fixing, and free-edge conditions, the identity $[L_h\mathbf{V}_n\cdot\mathbf{V}_q]_s=[\mathbf{V}_n\cdot L_h\mathbf{V}_q]_s$) is observed), then the following identities hold:

$$W_{nn}-\Omega_n I_{nn}=0,\quad W_{nq}=I_{nq}=0\ (n\neq q)$$
$$I_{nq}=2\rho h[\mathbf{V}_n\cdot\mathbf{V}_q]_s,\quad W_{nq}=[L_h\mathbf{V}_n\cdot\mathbf{V}_q]_s \tag{2.2}$$

the first of which expresses Clapeyron's theorem, while the second represents the orthogonality conditions in the theory of thin elastic shells.

We will seek the solution of (1.3) in the form of an expansion in eigenmodes (2.1):

$$\mathbf{u}=\sum_{q=1}^{\infty}A_q\mathbf{V}_q \tag{2.3}$$

Substituting (2.3) into (1.3), multiplying the resultant expression by V_n, and integrating over S, we obtain the following expression for the expansion coefficients in (2.3), with allowance for (2.2):

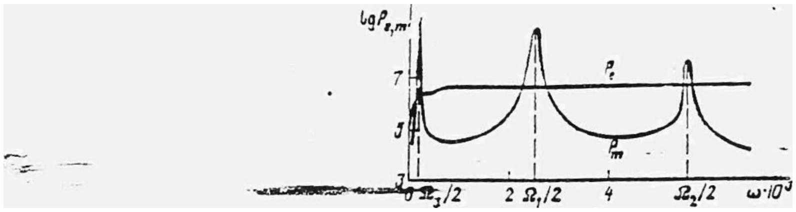
$$A_n=\{(\Omega_n^2-\Omega^2)+i\delta\Omega_n^2\}^{-1}[\mathbf{X}\cdot\mathbf{V}_n]_s/I_{nn} \tag{2.4}$$

Formulas (2.3) and (2.4) define the solution of Eqs. (1.3). We will use them and simplify expression (1.4) for the average power $\langle P_m\rangle$ of internal friction losses. Substituting X in the form of the left side of (1.3), replacing **u** in accordance with (2.3) and (2.4), and using identities (2.2), we obtain, after segregation of the imaginary part (taking $\bar{\mathbf{u}}=\Sigma\bar{A}_q\mathbf{V}_q$):

$$\langle P_m\rangle=\frac{1}{2}\sum_{n=1}^{\infty}\frac{\Omega\Omega_n^2\delta}{(\Omega_n^2-\Omega^2)^2+\delta^2\Omega_n^4}\frac{[\mathbf{X}\cdot\mathbf{V}_n]_s[\bar{\mathbf{X}}\cdot\mathbf{V}_n]_s}{2\rho h[\mathbf{V}_n^2]_s} \tag{2.5}$$

As we can see, now it is not necessary to solve (1.3) in order to determine $\langle P_m\rangle$. It is sufficient to know the eigensolutions of (2.1). Their properties and methods of determining them were described in [6].

We should note that, since the coefficient of internal friction $\delta \ll 1$, when twice the frequency of the inductor field coincides with any natural vibrational frequency of the shell, $2\omega = \Omega = \Omega_n$, resonance can occur (for $[\mathbf{X}\cdot\mathbf{V}_n]_s\neq 0$)), as well as the attendant sharp increase in $\langle P_m\rangle$. This was pointed out, e.g., in [1,2]. In this case we can confine ourselves to just one term in the sum of the series in (2.5). The dependence of this term on the frequency and on the coefficient of internal friction is determined by the multiplier $(\delta\Omega_n)^{-1}$, i.e., the intensity of the resonance "spike" in $\langle P_m\rangle$, generally speak-



ing, decrease as the frequency and coefficient of internal friction increase. The latter is due to a decrease in the amplitude of the resonances.

3. EXAMPLE OF CALCULATION

As an example, let us consider the problem of heating of an infinite cylindrical shell of radius R in a lateral magnetic field Bexp(iωt) with components (in cylindrical coordinates (z,θ,r), $\alpha_1=z/R$, $\alpha_2=\theta$, $\alpha_3=(r-R)/R$):

$$B_1=0,\quad B_2=B\sin\theta,\quad B_3=-B\cos\theta$$

The solution of the corresponding equations (1.1) has the form

$$\Phi=BR\frac{i\omega\beta}{1+i\omega\beta}\begin{Bmatrix}(r/R)^{-1}\\-(r/R)\end{Bmatrix}\cos\theta,\qquad \begin{Bmatrix}r\geqslant R+0\\r\leqslant R-0\end{Bmatrix}$$
$$F=2BRi\omega\beta/[(1+i\omega\beta)\cos\theta]\qquad(\beta=R/2\gamma)$$

In accordance with (1.2), the components of the magnetic-pressure vector X are

$$X_1=0$$
$$X_2=-\frac{B^2}{\mu_0}\frac{i\omega\beta}{(1+i\omega\beta)^2}\sin 2\theta,\qquad X_3=\frac{B^2}{\mu_0}\frac{i\omega\beta}{1+i\omega\beta}(\cos 2\theta-1)$$

In view of the orthogonal properties of the trigonometric functions, only three terms are retained in series (2.5), corresponding to eigensolutions $\Omega_1=C$, $\Omega_2=\sqrt{5}C$, $\Omega_3=\sqrt{60h}C$, $V_1^1=V_2^1=0$, $V_3^1=1$, $V_1^{2,3}=0$, $V_2^{2,3}=v_2^{2,3}\sin 2\theta$, $V_3^{2,3}=v_3^{2,3}\cos 2\theta$, $v_2^2=v_3^2=1$, $v_2^3=-v_3^3=1/2$, where $C=(E/\rho)^{1/2}$ is the speed of sound in the shell material.

Substituting the results into formulas (1.4) and (2.5) for $\langle P_e\rangle$ and $\langle P_m\rangle$ respectively, and integrating with respect to θ, we can derive the following formulas for determining the running heat-evolution power (per unit length of the shell):

$$\langle P_e\rangle=P_eB^2,\quad P_e=2\pi R\gamma\mu_0^{-1}\omega^2\beta^2(1+\omega^2\beta^2)^{-1}$$
$$\langle P_m\rangle=P_mB^4,\quad P_m=\frac{\pi R\delta\Omega}{4\mu_0^2\rho h}\sum_{n=1}^{3}\frac{a_n\Omega_n^2}{(\Omega_n^2-\Omega^2)^2+\Omega_n^4\delta^2}$$
$$a_1=2,\quad a_2=1/_3(9+\omega^2\beta^2)(1+\omega^2\beta^2)^{-1},\quad a_3=1/_3(1+4\omega^2\beta^2)(1+\omega^2\beta^2)^{-1}$$

The accompanying figure shows P_e and P_m, on a logarithmic scale, plotted versus ω for an aluminum cylinder with h = 10^{-3} m, R = 0.1 m, σ = $4\cdot10^7$ $(\text{ohm}\cdot\text{m})^{-1}$, ρ = 2.7·103 kg/m^3, E = $6.85\cdot10^{10}$ N/m^2, δ = $7\cdot10^{-3}$. As we can see, at the resonances ($\omega=\Omega_n/2$, $\Omega_1=5037$ sec^{-1}, $\Omega_2=11260$ sec^{-1}, $\Omega_3=390$ sec^{-1}) the power $\langle P_m\rangle$ of heat evolution as a result of internal friction displays spikes; for B = 1 T it predominates, by several orders of magnitude, over the eddy-current loss power (which remains virtually constant beginning at certain frequencies). As the magnetic induction of the field B increases, the degree of predominance increases quadratically.

REFERENCES

1. Ya. S. Pontryagin, Ya. I. Burak, and V. F. Kondrat, Magnetothermoelasticity of Electrically Conducting Solids [in Russian], Nauk. dumka, Kiev, 1982.

2. M. I. Kiselev, S. Yu. Ryzhkov, and S. V. Sobolev, "On the theory of magnetoacoustic heating of a conducting viscoelastic layer with fastened boundaries," in: Abstracts of Reports to Third All-Union Symposium on Theoretical Issues in Magnetoelasticity [in Russian], pp. 91-94, Erevan. un-t, Erevan, 1984.

3. A. L. Radovinskii, "Dynamics of elastic electrically conducting shells in stationary and nonstationary magnetic fields," PMM, vol. 50, no. 5, pp. 796-803, 1986.
4. E. S. Sorokin, On the Theory of Internal Friction for the Case of Vibration of Elastic Systems [in Russian], Gosstroiizdat, Moscow, 1960.
5. A. L. Radovinskii, "Equations of electromagnetic processes in thin conducting shells," Izv. AN SSSR. Energetika i transport, no. 4, pp. 164-170, 1987.
6. A. L. Gol'denveizer, V. B. Lidskii, and P. E. Tovstik, Free Vibrations of Thin Elastic Shells [in Russian], Nauka, Moscow, 1979.
7. O. V. Tozoni and I. D. Maergoiz, Calculation of Three-Dimensional Electromagnetic Fields [in Russian], Tekhnika, Kiev, 1974.
8. V. I. Astakhov, "Problem of calculating a quasi-stationary electromagnetic field in conducting shells," Izv. VUZ. Elektromekhanika, no. 1, pp. 15-29, 1985.

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