ON THE EQUATIONS FOR ELECTROMAGNETIC PROCESSES IN THIN CONDUCTING SHELLS

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Asymptotic integration (based on the smallness of the relative-thickness parameter) of the Maxwell equations with respect to the shell thickness is used to derive the differential form of the equations for the electromagnetic processes in thin conducting plates and shells. Analysis of the asymptotic error of the equations is used as a basis for determining the limits of their applicability on the basis of the variability of the processes considered in space and in time. The classes of problems that can be solved on their basis is determined and corresponding boundary and initial conditions are formulated. A theorem is derived for the power balance, together with a formula for determining the eddy-current losses in harmonic processes.

Problems involving the determination of eddy currents in thin plates and shells made of materials having finite electrical conductivity and located in the variable magnetic field of external sources are quite important in the development and design of components of electrical devices (magnetic shields of transformers and transmission lines), sensing elements in instrumentation fabrication (induction sensors, magnetic dampers), production processes (magnetic stamping, nondestructive inspection by the eddy-current method), etc.

The equations of [1,2], which rest on hypotheses based on an idealization of a shell, it being represented as an infinitesimally thin current layer, have made it possible to create very effective methods [3,4] for the solution of a broad class of problems. A survey of other studies pertaining to this problem is to be found in [1].

Obtaining equations of this sort represents a conventional problem in asymptotic integration of differential equations in thin regions. Methods have been developed for such a procedure, for example, in mechanics with application to derivation of two-dimensional equations in the theory of elastic shells from three-dimensional elasticity-theoretic equations [5].

Here we shall employ asymptotic integration (as in [5]) of the Maxwell equations over the thickness of a shell to derive the equations of the given problem, together with the corresponding boundary and initial conditions and the limits of applicability (in particular, with respect to the frequency of a harmonic process). The relationship of the results with approximate approaches in the low-frequency region is investigated. A power-balance theorem is formulated.

Formulation of problem. We shall assume that in an unbounded space V we are given a triorthogonal coordinate system (α_1, α_2, α_3) in which the coordinate surface $\alpha_3 = 0$ coincides with the midplane S of a certain shell occupying a region $V^{(i)}$ bounded by the face surfaces $\alpha_3 = \pm b$ and the closed edge surface $\phi(\alpha_1, \alpha_2) = 0$. We assume that the exterior region $V^{(e)} = V - V^{(i)}$ is occupied by a nonferromagnetic substance whose properties are identified with the properties of free space, while the interior region is filled by a material with finite constant electrical conductivity γ and relative permeability $\mu_r = \text{const}$.

Neglecting displacement currents and assuming that there are no external currents we write the Maxwell equations in the quasistationary approximation in the following form [6]:

$$-\Delta \mathbf{B}_\Sigma + \mu_r \mu_0 \gamma \frac{\partial \mathbf{B}_\Sigma}{\partial t} = 0, \quad \operatorname{div} \mathbf{B}_\Sigma = 0. \tag{1}$$

Here B_Σ is the total magnetic induction vector; t is the time; μ_0 is the magnetic constant; Δ is the Laplace operator.

We shall assume that

$$B_\Sigma = \begin{cases} B^{(e)} + b^{(e)} & \text{in } V^{(e)}, \\ B^{(i)} + b^{(i)} & \text{in } V^{(i)}, \end{cases} \tag{2}$$

where $B^{(e)}$, $B^{(i)}$ correspond to the induction specified by solution of the electrodynamics problem for a certain external source under the assumption that the shell is a dielectric having relative permeability μ_r.

We also introduce the concept of the induction B determined over the entire region V and obtained by solving the same problem under the assumption that there is no shell (the entire space has the properties of free space).

By definition $B, B^{(e)}, B^{(i)}$ satisfy the equations

$$\Delta\{\cdot\} = \operatorname{div}\{\cdot\} = \operatorname{curl}\{\cdot\} = 0, \quad \{\cdot\} = \{B, B^{(e)}, B^{(i)}\},$$

so that following the substitution of (2), we may write (1) in the form

$$\Delta b^{(e)} = 0, \quad \operatorname{div} b^{(e)} = 0 \quad \text{в} \quad V^{(e)}, \tag{3}$$

$$-\Delta b^{(i)} + \mu_r \mu_0 \gamma \frac{\partial}{\partial t}(b^{(i)} + B^{(i)}) = 0, \quad \operatorname{div} b^{(i)} = 0 \quad \text{в} \quad V^{(i)}. \tag{4}$$

Equations (3), (4) must be integrated over the entire region V with satisfaction at the face and edge surfaces of the conditions [6] requiring equality of the normal components of the magnetic induction, the tangential components of the magnetic field, and the requirement that there be no electric current along the normal to the surface. These are expressed by means of the equalities

$$(b^{(e)} - 1/\mu_r\, b^{(i)}) \times n = 0, \quad (b^{(e)} - b^{(i)}) \cdot n = 0, \quad n \cdot \operatorname{curl} b^{(i)} = 0, \tag{5}$$

where n is the vector (exterior with respect to $V^{(e)}$) representing the normal to the interface of regions $V^{(i)}$ and $V^{(e)}$.

The requirement of boundedness at infinity is additionally imposed on the induction $b^{(e)}$

Solving the problem, we may determine all the remaining parameters of the electromagnetic field by direct operations. In particular, the eddy current density j and the vector q representing the volume ponderomotive forces are expressed on the basis of the formulas

$$j = \frac{1}{\mu_r \mu_0} \operatorname{curl} b^{(i)}, \quad q = \frac{1}{\mu_r \mu_0} [\operatorname{rot} b^{(i)} \times (B^{(i)} + b^{(i)})]. \tag{6}$$

Asymptotic integration. In (4) we go over to the new independent variables ξ_k, ζ, τ, making use of the formulas

$$\alpha_k = \eta^p R \xi_k, \quad \alpha_3 = \eta R \zeta, \quad t = \eta^r \mu_r \mu_0 \sigma R^2 \tau, \tag{7}$$

where $\eta = h/R$ is the relative half-thickness; R is the characteristic radius of curvature of surface S; p, r are numbers to be determined subsequently.

We shall assume that the subscripts l, k, m employed take on the values $l = 1, 2, 3$; $k, m = 1, 2$; $k \neq m$.

We refer surface S to the lines of curvature. Then the Lame coefficients are determined by the formulas

$$H_k = A_k a_k, \quad a_k = 1 + \alpha_3/R_k, \quad H_3 = 1, \tag{8}$$

where R_k are the normal radii of curvature of surface S.

The various combinations of α_1, α_2 produced by their multiplication or division, occurring in the equations (following substitution of (8) into them) are replaced by the series

$$f(a_1, a_2) = \Sigma (f)_n (\eta\zeta)^n, \qquad (f)_n = \frac{1}{n!}\left[\frac{\partial^n f}{\partial(\eta\zeta)^n}\right]_{\zeta=0}. \tag{9}$$

We use Σ everywhere to indicate summation from n = 0 to n = ∞.

In determining the terms of the expansions (9) it is necessary to replace α_3 in (8) in accordance with (7). In particular, we then obtain

$$(f)_0 = 1, \quad (a_1 a_2)_1 = \frac{R}{R_1} + \frac{R}{R_2}, \quad (a_k)_1 = -\left(\frac{1}{a_k}\right)_1 = \frac{R}{R_k}, \quad (a_1 a_2)_2 = \frac{R^2}{R_1 R_2}.$$

We represent the components of the specified vector $B^{(1)}$ in the form of expansions:

$$B_l^{(1)} = B^{(1)} \Sigma (\eta\zeta)^n B_{ln}^{(1)}, \qquad B_{ln}^{(1)} = \frac{1}{B^{(1)}}\left[\frac{\partial^n B_l^{(1)}}{\partial \alpha_3^n}\right]_{\alpha_3=0}, \tag{10}$$

where $B^{(1)} = \max_s |B^{(1)}|$ is the quantity with which we shall compare the asymptotic forms for the eddy-current induction.

We shall seek the components of the vector $\mathbf{b}^{(1)}$ as

$$b_l^{(1)} = B^{(1)} \eta^c \Sigma \zeta^n \eta^{c_{ln}} b_{ln}, \tag{11}$$

where $b_{ln} = b_{ln}(\alpha_1, \alpha_2, t)$; c, c_{ln} are numbers whose values will be determined below.

We make the substitutions (7)-(11) in (4) and satisfy these equations by satisfying the relationships obtained by successive incrementation of the coefficients in them with identical powers of ζ (we call them the ζ-equations). Here we assume that the numbers p, r, c, c_{ln} are so chosen that differentiation of the unknown functions with respect to ξ_k, τ will not lead to an asymptotic change in them, while the b_{ln} values determined from the ζ-equations have order η^0. (This indicates, in particular, that the number p coincides in meaning with the variability index [5] for the unknown state and that r is the analog of the number introduced in [7] that reduces the asymptotic form of the frequency parameter). We assume that in accordance with (9) and (10) the quantitites $(f)_n$ and $B^{(1)}$ have order η^0. Then all coefficients occuring on b_{ln} in the ζ-equations will have structure $\eta^K P$, where P are certain operators or factors that do not affect the asymptotic form of the corresponding term; η^K is a factor determining its asymptotic order. Then in each ζ-equation we may isolate asymptotically smooth terms containing η to the smallest power. The ζ-equations obtained after discarding of secondary terms should be formally consistent [7], i.e., they should satisfy certain conditions, the first of which consists in the following: several different expressions should not be obtained for determination of the same quantity; the second requires that in certain of the ζ-equations the asymptotically smooth (preserved) terms should include, in addition to terms that contain b_{ln}, terms with $B_{ln}^{(1)}$.

These conditions are satisfied provided for the first terms of the expansions (11) we take

$$c_{l0} = c_{k1} = c_{k2} = 0, \quad c_{31} = 1, \quad c_{32} = 1 - p, \qquad c = \begin{cases} 1 + p - r, & r < 1 + p \\ 0, & r \geq 1 + p. \end{cases} \tag{12}$$

Discarding nothing, following identical transformations we obtain the following relationships from the ζ-equations obtained by equating the coefficients on ζ^0 and ζ in the second equation of (4) and on ζ^0 in the first:

$$b_{31} = -[(a_1 a_2)_1 b_{30} + \eta^{-p}(P_{01} b_{10} + P_{02} b_{20})],$$

$$b_{32} = -\tfrac{1}{2}\{(P_{01} b_{11} + P_{02} b_{21}) + \eta^{1+p}[(a_1 a_2)_1 b_{31} + (a_1 a_2)_2 b_{30}] + \eta(P_{11} b_{10} + P_{12} b_{20})\}, \tag{13}$$

$$b_{k2} = \tfrac{1}{2}\left[\eta^{2-c-r}\frac{\partial B_{k0}^{(1)}}{\partial \tau} - \eta (a_1 a_2)_1 b_{k1} - \eta^{2-2p} \Delta_k b_{k0} + \eta^{2-r}\frac{\partial b_{k0}}{\partial \tau}\right], \tag{14}$$

$$\eta^{c-1-p}(P_{01}b_{11}+P_{02}b_{21})+\eta^{c-p}(P_{11}b_{10}+P_{12}b_{20})+$$
$$+\eta^{c}\left[\eta^{-r}\frac{\partial}{\partial\tau}-\eta^{-2p}\Delta_{\xi}-(a_1a_2)_2\right]b_{30}=-\eta^{-r}\frac{\partial B_{30}^{(i)}}{\partial\tau}, \qquad (15)$$

where the operators

$$\Delta_{\xi}b_{10}=\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\xi_1}\left(\frac{A_2}{A_1}\frac{\partial b_{10}}{\partial\xi_1}\right)+\frac{\partial}{\partial\xi_2}\left(\frac{A_1}{A_2}\frac{\partial b_{10}}{\partial\xi_2}\right)\right],$$
$$P_{0k}b_{ln}=\frac{1}{A_1A_2}\frac{\partial}{\partial\xi_k}(A_mb_{ln}),\qquad P_{1k}b_{ln}=\frac{1}{A_1A_2}\left[\left(\frac{1}{a_k}\right)_1\frac{\partial}{\partial\xi_k}+\eta^{p}(a_m)_1\right](A_mb_{ln}).$$

We may use (12) to determine the mutual asymptotic form of any pair of terms in each of the equations (13)-(15). Below we shall construct equations accurate up to

$$O(\eta^{\varepsilon}),\quad \varepsilon=\min(2-r,\ 1-p), \qquad (16)$$

discarding terms of such order in comparison with asymptotically smooth terms. In comparison of the exponents in (13)-(15) we shall assume that p and r are bounded by the inequalities $0 \le p < 1$, $r < 2$, whose meaning will be indicated below.

Satisfaction of conditions at face surfaces. We satisfy (3), representing $b^{(e)}$ in the form

$$b^{(e)}=\operatorname{grad}\Phi=\frac{1}{H_1}\frac{\partial\Phi}{\partial\alpha_1}\cdot i_1+\frac{1}{H_2}\frac{\partial\Phi}{\partial\alpha_2}\cdot i_2+\frac{\partial\Phi}{\partial\alpha_3}\cdot i_3, \qquad (17)$$

where i_l are the unit vectors along α_l; Φ is the unknown potential function, satisfying the following equation in $V^{(e)}$:

$$\Delta\Phi=0. \qquad (18)$$

We take into account the fact that, in accordance with (11), (12), (17), and (9) the values of the $b^{(e)}$ and $b^{(i)}$ components at the face surfaces are expressed by the following equalities, to within quantitites of the order of (16):

$$b_3^{(i)\pm}=B^{(i)}\eta^{c}b_{30},\qquad b_k^{(i)\pm}=B^{(i)}\eta^{c}(b_{k0}\pm b_{k1}+b_{k2}),$$
$$b_3^{(e)\pm}=\left(\frac{\partial\Phi}{\partial\alpha_3}\right)^{\pm},\qquad b_k^{(e)\pm}=\frac{1}{A_k}\frac{\partial\Phi^{\pm}}{\partial\alpha_k}. \qquad (19)$$

Here the superscripts "+" and "-" indicate the values of the corresponding quantities when $\alpha_3 = +h$ and $\alpha_3 = -h$.

We satisfy the first two conditions of (17). They are equivalent to three scalar equalities which, Eqs. (19) being taken into account, may be reduced to the form

$$2B^{(i)}\eta^{c}b_{k1}=\frac{\mu_r}{A_k}\frac{\partial}{\partial\alpha_k}(\Phi^{+}-\Phi^{-}),\qquad B^{(i)}\eta^{c}b_{30}=\left(\frac{\partial\Phi}{\partial\alpha_3}\right)^{+}=\left(\frac{\partial\Phi}{\partial\alpha_3}\right)^{-}, \qquad (20)$$

$$2B^{(i)}\eta^{c}(b_{k0}+b_{k2})=\frac{\mu_r}{A_k}\frac{\partial}{\partial\alpha_k}(\Phi^{+}+\Phi^{-}). \qquad (21)$$

We note that (21) and (14) form a system of equations in the unknowns b_{k0} and b_{k2}. We represent b_{k2} from (21) and substitute in (14). Following identical transformations we obtain

$$2\eta^{c}b_{k0}=\frac{\mu_r}{B^{(i)}A_k}\frac{\partial}{\partial\alpha_k}(\Phi^{+}+\Phi^{-})+\eta^{c}\left[-\eta^{2-c-r}\frac{\partial B_{k0}^{(i)}}{\partial\tau}+\right.$$
$$\left.+\eta(a_1a_2)_1b_{k1}+\eta^{2-2p}\Delta_{\xi}b_{k0}-\eta^{2-r}\frac{\partial b_{k0}}{\partial\tau}\right].$$

Taking (12) into account it is not difficult to see that all terms that occur here in brackets may be discarded, with accuracy corresponding to (16), in comparison with the term on the left of the equals sign. This means that (21) may be replaced by the following expression, with the prescribed accuracy:

$$2B^{(i)}\eta^{\epsilon}b_{k0}=\frac{\mu_r}{A_k}\frac{\partial}{\partial\alpha_k}(\Phi^{+}+\Phi^{-}), \qquad (22)$$

while we make use of (14) to determine b_{k2}.

Since in accordance with (22) and (20) the functions b_{k0} and b_{k1} are components of the gradient of certain scalar functions on S, following substitution into it of the expansion (11) the last condition of (5) is satisfied with the prescribed accuracy (16).

Remark. Discarding of small terms when we go from (17) to the second pair of equalities in (19) actually means that when considering the Laplace equation (18) we reduce region $V^{(\pm)}$ to a mathematical section through S. Here the superscripts "+" and "-" mean that the corresponding quantities are taken when $\alpha_3 = +0$ and $\alpha_3 = -0$.

Solving equations. In (15) we make the inverse substitutions (7) and substitute $\eta^{\epsilon}b_{30}, \eta^{\epsilon}b_{k0}, \eta^{\epsilon}b_{k1}$ in accordance with (20) and (22). In (15) we simultaneously omit the second term associated with the factor $\eta^{\epsilon-p}$ as well as the second and third bracketed terms as compared with the first term that contains the factor $\eta^{\epsilon-1-p}$ (the error involved here is less than (16).) What is more, we take into account the fact that the first terms $B_{l0}^{(i)}$ of the expansions (10) may be expressed in terms of the first terms B_{l0} of the analogous expansions

$$B_l=\Sigma(\eta\zeta)^n B_{ln}, \quad B_{ln}=\left[\frac{\partial^n B_l}{\partial\alpha_3{}^n}\right]_{\alpha_3=0}$$

of the components of the vector B, since by definition, at the face surfaces they must satisfy conditions (similar to the first two conditions of (5)) that with the specified accuracy (16) reduce to the form

$$B^{(i)}B_{30}^{(i)}=B_{30}, \quad \frac{1}{\mu_r}B^{(i)}B_{k0}^{(i)}=B_{k0}. \qquad (23)$$

Going over in (15) to dimensioned quantities in accordance with (10), (11), and (23) and bringing in (18), (20) we obtain the following system of equations:

$$\Delta\Phi=0, \qquad (24)$$

$$\beta\Delta_s F+\frac{\partial f_3}{\partial t}=-\frac{\partial B_{30}}{\partial t},$$
$$F=\Phi^{+}-\Phi^{-}, \quad f_3=\left(\frac{\partial\Phi}{\partial\alpha_3}\right)^{+}-\left(\frac{\partial\Phi}{\partial\alpha_3}\right)^{-}, \qquad (25)$$

where $\beta=(2h\mu_0\gamma)^{-1}$; Δ_s is a two-dimensional Laplace operator on S; B_{30} is the value of the normal component of the vector B on S (the quantities B_{k0} subsequently utilized are simi- in meaning).

Following solution of (24), (25), where the function Φ becomes unknown, all terms of the expansions $b_{i(i)}$ may be determined by direct operations: b_{30}, b_{k0}, and b_{k1} from (20) and (22) and b_{31}, b_{32}, and b_{k2} from (13) and (14). **In particular we may determine the** tangential component of the electric current density, antisymmetric with respect to the midsurface (its principal part is determined by b_{k2}), arising (14) upon diffusion through the shell of the tangential components B_{k0} of the source magnetic field induction. This problem lies beyond the scope of the present paper. To determine the following terms of the expansions (11) it is necessary to construct the ζ-equations corresponding to (4), which follow those considered.

The condition for formal asymptotic conversion of the process considered consists in the requirement that the exponent ϵ of the small parameter η in (16), determining the asymptotic form of the discarded terms, be positive. In accordance with (7), (16) this yields the following symbolic expressions:

$$\frac{\partial}{\partial\alpha_i}<\eta^{-1}, \quad \frac{\partial}{\partial t}<\omega=(h^2\mu_r\mu_0\gamma)^{-1}, \qquad (26)$$

that bound the characteristics of the investigated processes by means of limits within which the variation of the unknown functions obtained by exact solution of the three-dimensional Maxwell equations over the shell thickness is weakly expressed and, in particular, within which there is no skin effect.

For processes that fail to satisfy (26) the given asymptotic process diverges and (24), (25) are inapplicable.

The linear current J in the shell and the magnetic pressure X acting on it are determined by integration of (6) over the thickness, following substitution into them of the expansions (11), (9) and discarding of asymptotically small terms. The formulas obtained in this manner have the following form:

$$J=\frac{1}{\mu_0}\left[-\frac{1}{A_2}\frac{\partial F}{\partial\alpha_2}\cdot i_1+\frac{1}{A_1}\frac{\partial F}{\partial\alpha_1}\cdot i_2\right],$$
$$X=\frac{1}{\mu_0}(B_{30}+f_3)\frac{1}{A_k}\frac{\partial F}{\partial\alpha_k}\cdot i_k-\frac{\mu_r}{\mu_0}(B_{k0}+f_k)\frac{1}{A_k}\frac{\partial F}{\partial\alpha_k}\cdot i_3, \qquad (27)$$

where $f_k=\frac{1}{2A_k}\frac{\partial}{\partial\alpha_k}(\Phi^+ + \Phi^-)$; summation is carried out over k = 1, 2.

Power balance. We consider the problem of the energy balance in our problem.

Let $U_1=\{\Phi, F, f_3\}$ be a certain solution of (24), (25). Then $U_2=\partial/\partial t\{\Phi, F, f_3\}$ will also be a solution. We write Green's theorem for the harmonic functions Φ and ∂Φ/∂t:

$$\frac{1}{2}\frac{\partial}{\partial t}\iiint_V (\mathrm{grad}\,\Phi)^2\,dv=\iint_{\Sigma+S^++S^-}\Phi\frac{\partial}{\partial n}\left(\frac{\partial\Phi}{\partial t}\right)ds \qquad (28)$$

in the region V included within the spherical surface Σ and containing the section of S with face surfaces S^+ and S^-. Taking into account the fact that on S^+ and S^- we have $\frac{\partial}{\partial n}=-\left(\frac{\partial}{\partial\alpha_3}\right)^+$ and $\frac{\partial}{\partial n}=+\left(\frac{\partial}{\partial\alpha_3}\right)^-$, and taking the definition (25) of the functions F and f_3 into account, we obtain

$$\iint_{S^++S^-}\Phi\frac{\partial}{\partial n}\left(\frac{\partial\Phi}{\partial t}\right)ds=-\iint_S F\frac{\partial f_3}{\partial t}\,ds. \qquad (29)$$

We multiply the first equation of (25) by F, integrate over S and, making use of Green's theorem on S, we obtain

$$\iint_S F\frac{\partial f_3}{\partial t}\,ds=-\iint_S F\frac{\partial B_{30}}{\partial t}\,ds+\beta\iint_S(\mathrm{grad}_s F)^2\,ds-\beta\oint_\Gamma F\frac{\partial F}{\partial N}\,dl, \qquad (30)$$

where grad_s is the surface gradient on S; N is the unit exterior normal to the edge of shell Γ on S.

Substituting (29), (30) into (28) and dividing all terms by μ_0, we obtain

$$\overbrace{\frac{1}{2h\mu_0^2\gamma}\iint_S(\mathrm{grad}_s F)^2\,ds}^{(1')}+\overbrace{\frac{1}{2\mu_0}\frac{\partial}{\partial t}\iiint_V(\mathrm{grad}\,\Phi)^2\,dv}^{(2')}=$$
$$=\overbrace{\frac{1}{\mu_0}\iint_S F\frac{\partial B_{30}}{\partial t}\,ds}^{(3')}+\overbrace{\frac{1}{2h\mu_0^2\gamma}\oint_\Gamma F\frac{\partial F}{\partial N}\,dl}^{(4')}+\overbrace{\frac{1}{\mu_0}\iint_\Sigma\Phi\frac{\partial}{\partial n}\left(\frac{\partial\Phi}{\partial t}\right)ds}^{(5')}. \qquad (31)$$

The given expression may be treated as a formalization of the power-balance theorem for the problem at hand. It consists in the following.

The power of the field of eddy currents in the shell (1') plus the power of the eddy-current field in the surrounding space (2') equal the sum of the power of the external magnetic field (3') and the powers lost owing to the outflow through the edge of the shell (4') and at infinity (5').

Boundary conditions. The last two terms (4') and (5') in (31) should equal zero, which determines the natural boundary conditions that must be annexed to (24), (25) when they are integrated.

The first of these conditions consists in satisfaction of one of the requirements that $F_\Gamma = 0$ or $\left.\frac{\partial F}{\partial N}\right|_\Gamma = 0$. The physical meaning of these requirements is readily understood from (27) for the linear current density. They respectively indicate equality to zero of the normal component of the electric current and the component tangential with respect to the shell edge and, consequently, determine the conditions for the isolated edge $(F|_\Gamma = 0)$ and the edge that is in contact with the ideal conductor $\left(\left.\frac{\partial F}{\partial N}\right|_\Gamma = 0\right)$.

The second condition corresponds to the conventional requirement that the potential Φ be bounded at infinity.

Eddy-current losses. With the aid of (31) it is not a complicated matter to determine the power lost. Its instantaneous value is determined by the set of terms occurring in (31) to the left of the equals sign and not dependent on the time. In the most common harmonic processes the power loss is

$$\frac{1}{2h\mu_0^2\gamma}\iint_s (\mathrm{grad}_s F \cdot \overline{\mathrm{grad}_s F})\, ds,$$

since the first term of (1) contains time differentiation $(\partial/\partial t(\mathrm{grad}\,\Phi \cdot \overline{\mathrm{grad}\,\Phi}) = 0)$.

On formulations of problems and solution methods. Let us look at some questions associated with the formulation of problems and techniques for the solution of Eqs. (24), (25).

In essence, Eqs. (24), (25) represent the differential form of the equations of [1], written in integral form.

This means, first, that they may be applied to solution of the problems formulated in [1]. By virtue of the expressions (26) obtained here, however, which bound the limits of applicability of the equations considered (in essence, with respect to [1] they indicate the limits within which the hypothesis that the current density is constant over the thickness is valid) the problem formulations should be refined, following which they may be stated in the following manner: we have the task of determining the eddy-current fields in harmonically varying magnetic fields for frequencies $\Omega < \omega = (h^2\mu_r\mu_0\gamma)^{-1}$, in fields whose values vary at the shell owing to relative motion (rotation) of the field and shell with linear velocity $v \ll R\omega$ in pulsed fields, both smooth (satisfying the condition $\partial/\partial t \ll \omega$) and step-type fields (for which $\partial/\partial t > \omega$ at the front). In the latter cases it is necessary to supplement (24), (25) with initial conditions: homogeneous $(\Phi, F, f_3)|_0 = 0$ for action of a smooth pulse and inhomogeneous $f_3|_0 = -B_{30}$ or $f_3|_0 = B_{30}$, respectively, with step connection or disconnection of a magnetic field having induction B. We note that with step connection or disconnection of a constant magnetic field, it is necessary to let $\partial B_{30}/\partial t = 0$ on the right side of the first equation of (25); the system of equations (24), (25) becomes homogeneous and its solution for the corresponding initial conditions will take the form of the sum of its characteristic solutions, exponentially damped in time.

Second, since (24), (25) may be reduced to (1), then the methods shown here may be used for their solution. Here the following comment is required. Methods for solving systems of equations resembling (24), (25) have been developed in the theory of hydroelasticity of thin plates and shells, and this is responsible for the broad possibilities for their application to solution of the problems considered here.

In particular, the question has been considered in [7] of the possibility of simplifying the equations of hydroelasticity by discarding asymptotically small terms. We may proceed in the same manner with (24), (25). The mutual asymptotic behavior of the terms on the left side of the first equation of (25) is specified by the powers of η for the corresponding terms in (15) (they take the form $\eta^{\varepsilon-1-\rho}$ and $\eta^{\varepsilon-r}$). For $\chi = 1+\rho-r > 0$ the second term on the left side of the first equation of (25) may be discarded, which involves an error η^χ. Then in place of (24), (25) we obtain $\Delta_s F = -\partial B_{30}/\partial t$, thus reducing the problem to a two-dimensional one. The condition $\chi > 0$ is equivalent to requiring that the characteristic linear velocity of perturbations not exceed the value $v' = (h\mu_r\mu_0\gamma)^{-1}$. In particular, for harmonic processes this condition takes the form $\Omega < \omega' = v'/R$, bounding the region of applicability of methods [8] based on neglecting the magnetic field of the eddy currents as compared with the magnetic field of the external source. We note that the illegitimacy of using the methods of [8] at high frequencies is confirmed by the numerical results cited in [3].

In conclusion it should be noted that the question of the admissibility of solution of the given problem in quasistationary approximation is similar to the question of neglecting allowance for the compressibility of a fluid in the hydroelasticity of shells. Questions pertaining to analysis of the error associated with this fall beyond the scope of this paper. Some of these questions have been considered in [9,10], and on the basis of the results reported their it may be stated that the error introduced by the quasistationarity of the initial equations is many orders less than the error (16) tolerated in derivation of (24), (25) themselves.

CONCLUSIONS

1.. The problem of determining the magnetic potential of a field of eddy currents appearing owing to the diffusion of an electromagnetic field into a thin shell of finite conductivity has been considered. It has been shown that it may be reduced to solution of a three-dimensional Laplace equation in the surrounding medium with an inhomogeneous mixed differential boundary condition at a mathematical section that coincides with the midsurface of the shell.

2. Utilization of the given method and similar ones is restricted to investigation of electromagnetic fields such that no skin effect appears on the thickness when they diffuse into a shell.

3. The power balance theorem obtained for the electromagnetic field of eddy currents may be utilized in developing variational techniques for solution of the problem. A formula is given for calculating the eddy-current losses for a harmonic process.

4. In the low-frequency region the problem may be reduced to a two-dimensional one solvable in the metric of the shell.

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