NONAXISYMMETRIC VIBRATIONS OF SHELLS OF REVOLUTION CONTAINING A FLUID

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In this paper we will consider linear harmonic nonaxisymmetric vibrations of thin fluid-filled shells of revolution that are closed at the vertex. We will employ approximate methods analogous to those used in [1,2] in the statics and dynamics of shells.

1. The position of the points of the shell and of some adjacent layer of fluid (wall layer) will be defined by the radius vector

$$\mathbf{M}(\alpha, \theta, \gamma) = \mathbf{P}(\alpha, \theta) + \gamma \mathbf{n} \tag{1.1}$$

Here $\mathbf{P}(\alpha, \theta)$ is the radius vector of the points of the center surface S of the shell; $\mathbf{n}$ is the unit vector of the inner normal to S; α, θ are the parameters of the geographical coordinate system specified on S; γ is a coordinate that increases along $\mathbf{n}$; and on S we have $\gamma = 0$.

We take the characteristic radius R^0 of S as the unit of length, and we will assume that in the wall layer $\gamma/R^0 \ll 1$.

The components U_i ($i = 1, 2, 3$) of the displacement vector of the center surface of the shell and the displacement potential Ψ of the fluid will be sought in the form

$$\begin{aligned} U_{1,3}(\alpha, \theta, t) &= u_{1,3}(\alpha) \cos m\theta \exp(i\omega t) \\ U_2(\alpha, \theta, t) &= u_2(\alpha) \sin m\theta \exp(i\omega t) \\ \Psi(\alpha, \theta, \gamma, t) &= \Phi(\alpha, \gamma) \cos m\theta \exp(i\omega t) \end{aligned} \tag{1.2}$$

where m is a specified number of waves along the parallel. We will employ the following equations as our starting-point.

After separation of variables, the equations of motion of the shell in contact with the fluid [3] can be brought to the form

$$\begin{gathered} (h^2 N_{ij} + L_{ij}) u_j + \lambda (1 - \nu^2) u_i \delta_{3i} \frac{1-\nu^2}{2h} \frac{\rho}{\rho_0} \lambda \Phi_* = 0 \\ \lambda = \rho_0 \omega^2 R^{02}/E \qquad (i, j = 1, 2, 3) \end{gathered} \tag{1.3}$$

where λ is a frequency parameter; h, E, ν are the half-thickness, Young's modulus, and Poisson's ratio of the shell material; ρ and ρ_0 are the densities of the fluid and the shell material; δ_{3j} is the Kronecker delta; the asterisk here and there henceforth denotes that the value of the corresponding function on S is being taken; L_{ij} and N_{ij} are the momentless and moment operators of the theory of shells, part of which are written below, while the rest are given in [4]:

$$L_{11} = \frac{1}{A^2} \frac{d^2}{d\alpha^2} + \frac{1}{A^2} \left(\frac{B'}{B} - \frac{A'}{A} \right) \frac{d}{d\alpha} + \left[\frac{B''}{B} - \frac{A'B'}{AB} - \left(\frac{B'}{B} \right)^2 \right] - \frac{1-\nu}{R_1 R_2} - \frac{1-\nu}{2} \frac{m^2}{B^2}$$

$$L_{12} = \frac{1+\nu}{2} \frac{m}{B} \frac{1}{A} \frac{d}{d\alpha} - \frac{3-\nu}{2} \frac{B'}{AB} \frac{m}{B}$$

$$L_{13}=\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)\frac{1}{A}\frac{d}{d\alpha}-\left(\frac{R_1'}{R_1^2}+\frac{R_2'}{R_2^2}\right)\frac{1}{A}, \quad L_{21}=-\frac{1+\nu}{2}\frac{m}{B}\frac{1}{A}\frac{d}{d\alpha}-\frac{3-\nu}{2}\frac{B'}{AB}\frac{m}{B}$$

$$L_{23}=L_{32}=-\left(\frac{1}{R_2}+\frac{\nu}{R_1}\right)\frac{m}{B}$$

$$L_{22}=\frac{1-\nu}{2}\frac{1}{A^2}\frac{d^2}{d\alpha^2}+\frac{1-\nu}{2}\frac{1}{A^2}\left(\frac{B'}{B}-\frac{A'}{A}\right)\frac{d}{d\alpha}+$$

$$+\frac{1-\nu}{2}\left[\frac{B''}{B}-\frac{A'B'}{AB}-\left(\frac{B'}{B}\right)^2\right]+\frac{1-\nu}{R_1R_2}-\frac{m^2}{B^2}$$

$$L_{31}=-\frac{1}{A}\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)\frac{d}{d\alpha}-\frac{1}{A}\left[\frac{B'}{B}\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)+\frac{1-\nu}{R_2^2}R_2'\right],$$

$$L_{33}=-\left(\frac{1}{R_1^2}+\frac{1}{R_2^2}+\frac{2\nu}{R_1R_2}\right)$$

$$N_{33}=-\frac{1}{3AB}\left(\frac{d}{d\alpha}\frac{B}{A}\frac{d}{d\alpha}-\frac{A}{B}m^2\right)\frac{1}{AB}\left(\frac{d}{d\alpha}\frac{B}{A}\frac{d}{d\alpha}-\frac{A}{B}m^2\right)$$

where A, B and R_1, R_2 are, respectively, the coefficients of the first quadratic form and the principal radii of curvature of S; here and henceforth the prime denotes derivatives with respect to α.

The equation of motion of a compressible fluid:

$$\Delta\Psi-\frac{1}{C^2}\frac{d^2\Psi}{dt^2}=0 \tag{1.4}$$

where Δ is the Laplace operator and C is the speed of sound in the fluid.

The (α, θ, γ) coordinate system is triorthogonal [1]. Therefore, the Laplace operator in the well layer can be written as follows:

$$\Delta=\frac{1}{H_1H_2H_3}\left[\frac{\partial}{\partial\alpha}\left(\frac{H_2H_3}{H_1}\frac{\partial}{\partial\alpha}\right)+\frac{\partial}{\partial\theta}\left(\frac{H_1H_3}{H_2}\frac{\partial}{\partial\theta}\right)+\frac{\partial}{\partial\gamma}\left(\frac{H_1H_2}{H_3}\frac{\partial}{\partial\gamma}\right)\right] \tag{1.5}$$

$$H_1=A(1-\gamma/R_1), \quad H_2=B(1-\gamma/R_2), \quad H_3=1$$

Substituting (1.2) and (1.5) into (1.4), and separating the variables θ and t, we obtain an equation in Φ:

$$\left\{\frac{\gamma_2}{\gamma_1}\frac{B}{A}\frac{\partial^2}{\partial\alpha^2}+\left[\frac{\gamma_2}{\gamma_1}\left(\frac{B}{A}\right)'-\gamma\frac{B}{A}\frac{\gamma_2R_1'/R_1^2-\gamma_1R_2'/R_2^2}{\gamma_1^2}\right]\frac{\partial}{\partial\alpha}-\frac{\gamma_1}{\gamma_2}\frac{A}{B}m^2+\right.$$

$$\left.+AB\gamma_1\gamma_2\frac{\partial^2}{\partial\gamma^2}-AB\left(\frac{1}{R_1}+\frac{1}{R_2}-\frac{2\gamma}{R_1R_2}\right)\frac{\partial}{\partial\gamma}+AB\gamma_1\gamma_2\lambda\left(\frac{C_0}{C}\right)^2\right\}\Phi=0 \tag{1.6}$$

$$C_0=\sqrt{E/\rho_0}, \quad \gamma_1=1-\gamma/R_1, \quad \gamma_2=1-\gamma/R_2$$

The no-flow condition should be observed on S:

$$\partial\Phi/\partial\gamma|_*=-u_3 \tag{1.7}$$

the difference between the center and inner face surfaces being disregarded in this case

2. We will seek the integrals of (1.3), (1.6), and (1.7) in the form

$$u_1=k^{-1}\xi e^{kf}, \quad u_2=k^{-2}\eta e^{kf}, \quad u_3=\zeta e^{kf}, \quad \Phi=k^{-1}\varphi e^{kF} \tag{2.1}$$

$$p=p_0+k^{-1}p_1+\ldots, \quad p_0\neq0 \quad (p=\xi, \eta, \zeta, \varphi), \quad k=h^{-1/2}[3(1-\nu^2)]^{1/4} \tag{2.2}$$

By analogy with [1], we call $f(\alpha)$ and $F(\alpha, \gamma)$ variability functions and $\xi(\alpha)$, $\eta(\alpha)$, $\zeta(\alpha)$ and $\varphi(\alpha, \gamma)$ intensity functions; $\xi_i(\alpha)$, $\eta_i(\alpha)$, $\zeta_i(\alpha)$, $\varphi_i(\alpha, \gamma)$ $(i=0, 1, 2, \ldots)$ are the expansion coefficients of the intensity functions.

We will consider quasi-transverse vibrations, i.e., vibrations for which $u_3 \gg (u_1, u_2)$ (here u_i are understood to mean the maximum values). We will assume that the number of waves along the parallel is small, and that the condition $m^2 \ll k^2$ is observed. Such vibrations are characterized [2] by $u_2 < u_1$, and hence the choice of expressions (2.1).

In asymptotic integration of the equations, it is necessary to take account of the orders of the coefficients that appear in them. Since the values of the coefficients of (1.3) and (1.6) that are specified in calculations may differ substantially in different cases, we introduce the parameters r, a, and b in accordance with the expressions

$$\lambda = h^{2r}, \quad \rho/\rho_0 = h^a, \quad C/C_0 = h^b \tag{2.3}$$

and we will assume for the time being that $r=0$, $a=1/2$, $b=1/4$.

It is shown in §6 that, given these a, b, and r values, the results are of a fairly general nature and can be extended to other combinations of numbers a, b, and r (also given there). We should note that the values of coefficients (2.3) corresponding to the given r, a, and b correspond to the case of vibrations of a water-filled shell with h = = 0.01 at frequencies for which $\lambda = O(1)$.

3. We separate out the variable γ, using Eq. (1.6) and the no-flow condition (1.7).

We write functions $F(\alpha, \gamma)$ and $\varphi_i(\alpha, \gamma)$ in the wall layer in the form of Taylor series expansions:

$$(F, \varphi_i) = \sum_{n=0}^{\infty} \gamma^n (F_n, \varphi_{in}) \tag{3.1}$$

where $F_n(\alpha)$, $\varphi_{in}(\alpha)$ are functions to be determined.

We will satisfy the no-flow condition by substituting expressions (2.1) and (3.1) into (1.7) and equating the exponents and coefficients for equal descending powers of k. We obtain the equations

$$F_0 = f, \quad \zeta_0 = -F_1\varphi_{00}\{k^0\}, \quad \zeta_1 = -F_1\varphi_{10} - \varphi_{01}\{k^{-1}\} \tag{3.2}$$

Here and henceforth the braces indicate quantities whose coefficients are equated to obtain the given equation.

Substituting (2.1) and (3.1) into (1.6), representing the coefficients of (1.6) as series in γ/R_1 and γ/R_2, and setting the multipliers for $k^i\gamma^j$ equal to zero, we obtain the chain of equations

$$\begin{gathered}
\left(\frac{f'}{A}\right)^2 + F_1^2 = 0 \quad \{k^2\gamma^0\} \\
-\frac{2}{R_1}\left(\frac{f'}{A}\right)^2 + 2\frac{F_1'f'}{A^2} + 4F_1F_2 = 0 \quad \{k^2\gamma\} \\
2\frac{f'}{A^2}\varphi_{00}' + \frac{f'}{A^2}\left(\frac{f''}{f'} + \frac{B'}{B} - \frac{A'}{A}\right)\varphi_{00} + 2F_1\varphi_{01} + \\
+2F_2\varphi_{00} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right)F_1\varphi_{00} + \mu\varphi_{00} = 0 \quad \{k\gamma^0\}, \quad \mu = \frac{\lambda}{k}\left(\frac{C_0}{C}\right)^2
\end{gathered} \tag{3.3}$$

Here it was taken into account that $m^2 \ll k^2$, and, in accordance with (2.3) and (2.4), $\mu = O(h^0)$.

Equations (3.2) (for ζ_0 and ζ_1) and (3.3) make it possible to represent the potential Φ on S that appears in (1.3) as follows:

$$\begin{aligned}
\Phi_* = -\frac{k^{-1}}{F_1}\Big\{\zeta_0 + k^{-1}\Big[\zeta_1 - \frac{\zeta_0'}{f'} - \left(\frac{A'}{A} + \frac{1}{2}\frac{B'}{B} - \frac{f''}{f'}\right)\frac{\zeta_0}{f'} - \\
-\left(\frac{1}{R_2} - \frac{\mu}{F_1}\right)\frac{\zeta_0}{2F_1}\Big] + O(k^{-2})\Big\}e^{kf}
\end{aligned} \tag{3.4}$$

where function F_1 can be expressed in terms of f' in accordance with (3.3).

4. Now the technique for integrating (1.3) differs little from the familar ones of [1,2].

Substituting (2.1) and (3.4) into (1.3), and setting the coefficients for descending powers of k equal to zero, we obtain equations in the unknown functions f, ξ_i, η_i, ζ_i.

The homogeneous algebraic system in ξ_0, η_0, ζ_0, that is obtained by equating the coefficients for the higher degrees of k, has the nontrivial solutions

$$\xi_0=-\frac{A}{f'}\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)\zeta_0, \quad \eta_0=-\frac{m}{B}\left(\frac{A}{f'}\right)^2\left(\frac{1}{R_1}-\frac{2+\nu}{R_2}\right)\zeta_0 \tag{4.1}$$

provided that the following equations are satisfied:

$$\Omega(x)=x^3+(R_2^{-2}-\lambda)x+\delta\lambda=0, \quad \delta=\rho/(2hk\rho_0) \tag{4.2}$$

$$(f')^2=-A^2X^2, \quad F_1=X \tag{4.3}$$

where X is the negative root of (4.2) (the choice of the root is discussed below)

The compatibility condition for the equations obtained by setting the coefficients for the subsequent powers of k equal to zero can be expressed in the form of a linear first-order differential equation:

$$\frac{\zeta_0'}{\zeta_0}+\frac{1}{2}\frac{B'}{B}-\frac{1}{2}\frac{X'}{X}-\frac{1}{2}\frac{\Omega_1'}{\Omega_1}-\frac{\delta\lambda f'}{2\Omega_1X^2}\left(\frac{1}{R_2}-\frac{\mu}{X}\right)=0 \tag{4.4}$$

$$\Omega_1=\partial\Omega/\partial x|_{x=X}$$

Integrating (4.4), we obtain (taking account of of (4.1)) the first expansion terms of the intensity functions.

The choice of the root of (4.2) predetermines the obtaining of particular integrals of (1.3), (1.6), and (1.7).

Here we will consider only integrals (2.1), for which

$$\mathrm{Re}(f)=0, \quad \mathrm{Im}(f)\neq 0, \quad \mathrm{Re}(X)<0 \tag{4.5}$$

These integrals oscillate on S along the meridian, this being ensures by satisfaction of the first two conditions in (4.5), and exponentially attenuate in the fluid along the normal to the shell. This last circumstance follows from the third condition in (4.5), the second equation in (4.3), and the expression

$$\Phi(\alpha,\gamma)=\Phi_*\exp\{k[F_1\gamma+O(\gamma^2)]\} \tag{4.6}$$

obtained upon substituting (3.1) into (2.1)

The vibrations corresponding to these integrals are localized in the wall layer of the fluid. Therefore, the remaining part of the region occupied by the fluid can be disregarded in the initial approximation.

Analysis of (4.2) ($\Omega(-\infty)=-\infty$, $\Omega(0)=\delta\lambda>0$) shows that there is always one negative real root $X<0$. Using Descartes' rule, we can show that Eq. (4.2) has no other negative roots.

The choice of sign in the roots of the first equation in (4.3) determines two different solutions of system (4.1)-(4.4) that satisfy conditions (4.5). Substituting these solutions into (2.1), and taking account of (3.4) and (4.6), we obtain the following integrals of system (1.3), (1.6), (1.7):

$$\begin{gathered}
u_1=k^{-1}\frac{\chi}{X}\left(\frac{1}{R_1}+\frac{\nu}{R_2}\right)\sin(kM+L+\beta)\\
u_2=k^{-2}\frac{m}{B}\frac{\chi}{X^2}\left(\frac{1}{R_1}-\frac{1+\nu}{R_2}\right)\cos(kM+L+\beta)\\
u_3=\chi\cos(kM+L+\beta), \quad \Phi=\Phi_* e^{kX\gamma}\\
\Phi_*=-k^{-1}\frac{\chi}{X}\cos(kM+L+\beta)\\
M=-\int XA\,d\alpha, \quad L=-\frac{\delta\lambda}{2}\int\left(\frac{1}{R_2}-\frac{\mu}{X}\right)\frac{A\,d\alpha}{\Omega_1X}, \quad \chi=\left[-\frac{X}{B\Omega_1}\right]^{1/2}
\end{gathered} \tag{4.7}$$

where β is an arbitrary constant.

5. The method that yields integrals (4.7) becomes invalid as we approach the vertices of the shell, where B = 0 and the intensity functions increase without limit.

Let us confine ourselves to shells with vertices (of cupola type), in the neighborhood of which surface S can be replaced by a sphere ($R_1 = R_2 = R$) with some degree of accuracy.

In the neighborhood of any of these vertices, we introduce a (1, 1) coordinate system [2], when $A=1, B=\alpha+\alpha^3/6R^2+O(\alpha^5)$.

In the neighborhood of the vertex, we will seek the deflection of the shell and the displacement potential of the fluid in the form

$$U_3(\alpha,\theta,t)=W(\alpha)\cos m\theta e^{i\omega t},\quad \Psi(\alpha,\theta,\gamma,t)=\psi(\alpha,\gamma)\cos m\theta e^{i\omega t},$$
$$\psi(\alpha,\gamma)=Z(\alpha)Y(\gamma) \tag{5.1}$$

Substituting (5.1) into (1.4) and (1.5) and separating variables, we obtain the equations

$$\frac{d^2Y}{d\gamma^2}-\frac{2}{R-\gamma}\frac{dY}{d\gamma}+\left[\lambda\left(\frac{C_0}{C}\right)^2-\frac{k^2\varkappa^2R^2}{(R-\gamma)^2}\right]Y=0 \tag{5.2}$$

$$\Delta_m Z+k^2\varkappa^2Z=0,\quad \Delta_m=\frac{d^2}{d\alpha^2}+\frac{B'}{B}\frac{d}{d\alpha}-\frac{m^2}{B^2} \tag{5.3}$$

where $k^2\varkappa^2$ is the separation parameter.

In the case under consideration, Eqs. (1.3) can be brought to the form

$$\frac{h^2}{3(1-\nu^2)}\Delta_m\Delta_m\Delta_m W+(R^{-2}-\lambda)\Delta_m W-\frac{1}{2h}\frac{\rho}{\rho_0}\lambda\Delta_m\psi_*=0 \tag{5.4}$$

Here we employ a method analogous to that of [4] for deriving the equations of force vibrations of shallow spherical shells in a vacuum. As in [4], we disregard the effect of tangential forces of inertia of the shell and the contribution of tangential displacements to the change in curvatures.

In the notation of (5.1), the no-flow condition has the form

$$\partial\psi/\partial\gamma|_*=-W \tag{5.5}$$

Assuming that $|\varkappa|=O(1)$, $\varkappa<0$, and taking account of (2.4), we write the solution of (5.2), bounded in the region occupied by the fluid ($0\leqslant\gamma\leqslant R$), in the form

$$Y=(1-\gamma/R)^{-k\varkappa R} \tag{5.6}$$

In a small neighborhood of the vertex ($(\alpha/R)^2\ll1$) we will assume that $B=\alpha$, and we replace operator Δ_m in accordance with the following:

$$\Delta_m=\Delta_m'=\frac{d^2}{d\alpha^2}+\frac{1}{\alpha}\frac{d}{d\alpha}-\frac{m^2}{\alpha^2} \tag{5.7}$$

Then Eq. (5.3) formally coincides with an m-th order Bessel equation and will have the solution $Z=J_m(-k\varkappa\alpha)$. We determine function W by substituting ψ, with allowance for (5.1) and (5.6), into no-flow condition (5.5). Thus, we obtain

$$W=-k\varkappa J_m(-k\varkappa\alpha),\qquad \psi=(1-\gamma/R)^{-k\varkappa R}J_m(-k\varkappa\alpha) \tag{5.8}$$

Making substitutions (5.7) and (5.8) in (5.4), and eliminating the multiplier $k^2\varkappa J_m(-k\varkappa\alpha)\neq0$, we obtain an equation in $\varkappa$:

$$\varkappa^2+(R^{-2}-\lambda)\varkappa+\delta\lambda=0 \tag{5.9}$$

which formally coincides with (4.2) and also always has one and only one negative root.

6. We will join the resultant integrals (4.7) and (5.8) on the basis of the deflection functions of the shell and the displacement potential of the fluid on S. The latter is permissible in view of the rapid attenuation of the displacement potential along the normal to S.

Using the properties of Bessel functions, for $-k\varkappa\alpha\gg1$ we write (5.8) in the form

$$W = -k\varkappa\psi_* \approx \left[-\frac{2k\varkappa}{\pi\alpha}\right]^{1/2} \cos\left(-k\varkappa\alpha - \frac{m\pi}{2} - \frac{\pi}{4}\right). \tag{6.1}$$

Within the framework of the assumptions made in §5, we can assume in the neighborhood of the vertex that we have $R_1 = R_2 = R$, $A = 1$, $B = \alpha$. Then it is evident from comparing (4.2) and (5.9) that $X = \varkappa$, while the other expressions are as follows:

$$M = -\varkappa\alpha,\; L = \frac{\delta\lambda\alpha}{2\varkappa\Omega_{1\varkappa}}\left(\frac{1}{R} - \frac{\lambda\mu}{\varkappa}\right),\; \chi = \left[-\frac{\varkappa}{\alpha\Omega_{1\varkappa}}\right]^{1/2},\; \Omega_{1\varkappa} = \Omega_1|_{X=\varkappa}^{R_2=R}$$

Thus, integrals (4.7) can be written as follows as $\alpha \to 0$:

$$u_3 = -k\varkappa\Phi_* = \left[-\frac{\varkappa}{\Omega_{1\varkappa}\alpha}\right]^{1/2} \cos\{-[k\varkappa + O(1)]\alpha + \beta\} \tag{6.2}$$

Comparison of (6.1) and (6.2) indicates that as $\alpha \to 0$:

$$(u_3\Phi_*) \to [\pi/2k\Omega_{1\varkappa}]^{1/2}(W,\ \psi_*) \quad \text{for} \quad \beta = -m\pi/2 - \pi/4$$

i.e., integrals (4.7) can be continued in regular fashion to the vertex with coordinate $\alpha = \alpha_i$ for

$$M(\alpha_i, \alpha) = -\int_{\alpha_i}^{\alpha} XA\, d\alpha, \quad L(\alpha_i, \alpha) = -\frac{\delta\lambda}{2}\int_{\alpha_i}^{\alpha}\left(\frac{1}{R_2} - \frac{\mu}{X}\right)\frac{A\, d\alpha}{\Omega_1 X} \tag{6.3}$$

$$\beta = -m\pi/2 - \pi/4$$

To determine the frequencies and modes of natural vibrations of the shell with vertex coordinates α_1 and α_2, we choose some point α_0 of the meridian $(\alpha_1 < \alpha_0 < \alpha_2)$ and set up integrals (4.7)-(6.3) separately for $\alpha_0 \leqslant \alpha \leqslant \alpha_2$ and for $\alpha_1 \leqslant \alpha \leqslant \alpha_0$. The joining condition for these integrals for $\alpha = \alpha_0$ yields the frequency equation

$$\sin[kM(\alpha_1, \alpha_2) + L(\alpha_1, \alpha_2) + 2\beta] = 0 \tag{6.4}$$

The frequency parameter λ appears implicitly in (6.4), in terms of the integrands of (6.3), which generally depend on α and λ. In the general case, frequency equation (6.4) must be solved by numerical methods.

The vibrational modes corresponding to the resultant frequencies are defined by (4.7)-(6.3) everywhere on the shell except for the vertices, where formulas (5.8) should be employed.

Up to now it has been assumed that the parameters a, b, and r introduced in (2.3), satisfy the conditions $r = 0$, $a = 1/2$, $b = 1/4$.

For different values of a, b, and r, the root of (4.2) can be written as follows:

$$X = h^{1/2 - p} q, \quad q = O(h^0) \tag{6.5}$$

Here p is the variability index [1,2] of the integrals obtained in §4; this can be established by substituting (2.2) and (6.5) into (4.7) and allowing for the fact that $k = O(h^{1/2})$, and hence $kM = O(h^{-p})$.

The use of formulas (4.7)-(6.3) and (6.4) for different values of a, b, and r is justifiable if, first, integrals (4.7) possess a large variability, i.e., if $p > 0$; and, second, if there is substantiation for the simplifications made in §§3, 4 in deriving the equations for determining the intensity and variability functions, these simplifications involving the disregard of asymptotically small terms. In verifying the second condition, it should be borne in mind that, in accordance with (4.3) and (6.5),

$$F_1 = O(h^{1/2 - p}), \quad f' = O(h^{1/2 - p}).$$

It can be established that the conditions formulated here are satisfied if

$$0 \leqslant a \leqslant \frac{1}{2}, \quad b < \min\left(\frac{1-a}{2}, a\right), \quad -1 + 2b < r < \frac{1-a}{2} \tag{6.6}$$

Thus, inequalities (6.6) define the region of applicability of our results. The resultant integrals possess variability indexes $0<p<1-b$.

If we fix the frequencies under consideration (i.e., assume some r value within lim (6.6)), then we can weaken constraints (6.6) on the permissible values of a and b, as done in §2, where $b=\min(1/2(1-a),\ a)=1/4$.

For $-1<r\leqslant-1+2b$, the calculations that led in §3 to the determination of Φ_* are not valid. However, we will note, without proof, that in this range of frequencies for $b<a$ we can employ the results of [2] obtained for vibrations of shells in vacuum. The corresponding frequency equation and the expression for the displacements u_1, u_2, u_3 follow formally from (6.3)-(6.4) for

$$L=0,\ \Omega(x)=x^3-\lambda x,\quad \Omega_1=\partial\Omega/\partial x|_{x=X}$$

if we assume that X is a negative root of the equation $\Omega=0$. These vibrations posses variability indexes $1-b\leqslant p<1$.

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